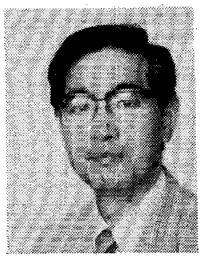
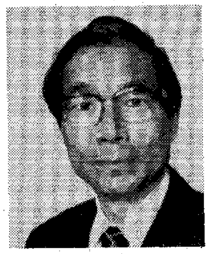


Hiroshi Shigesawa (S'62–M'63) was born in Hyogo, Japan, on January 5, 1939. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Doshisha University, Kyoto, Japan, in 1961, 1963, and 1969, respectively.

Since 1963, he has been with Doshisha University. From 1979 to 1980, he was a Visiting Scholar at the Microwave Research Institute, Polytechnic Institute of New York, Brooklyn, NY. Currently, he is a Professor of the Faculty of Engineering, Doshisha University. His present research activities involve microwave and submillimeter-wave transmission lines and devices of open structure, fiber optics, and scattering problems of electromagnetic waves.

Dr. Shigesawa is a member of the Institute of Electronics and Communication Engineers (IECE) of Japan, the Japan Society of Applied Physics, and the Optical Society of America (OSA).



Kei Takiyama (M'58) was born in Osaka, Japan, on October 20, 1920. He received the B.S. and Ph.D. degrees in electrical engineering from Kyoto University, Kyoto, Japan, in 1942 and 1955, respectively.

Since 1954, he has been a Professor of Electronic Engineering at Doshisha University, Kyoto, Japan, where he carried out research in the fields of microwave transmission lines and optical engineering. From 1957 to 1958, he was a Fulbright Scholar and a Research Associate at the Microwave Research Institute, Polytechnic Institute of Brooklyn, New York.

Dr. Takiyama is a member of the Institute of Electronics and Communication Engineers (IECE) of Japan, the Institute of Electrical Engineers of Japan, and the Optical Society of America (OSA).

Analysis of Hybrid Field Problems by the Method of Lines with Nonequidistant Discretization

HEINRICH DIESTEL AND STEPHAN B. WORM

Abstract —**The method of lines, which has been proved to be very efficient for calculating the characteristics of one-dimensional and two-dimensional planar microwave structures, is extended to nonequidistant discretization. By means of an intermediate transformation it is possible to maintain all essential transformation properties that are given in the case of equidistant discretization. The flexibility of the method of lines is increased substantially. As a consequence, the accuracy is improved with reduced computational effort.**

I. Introduction

A SUCCESSFUL DESIGN of planar microwave circuits presupposes accurate knowledge of the characteristics of the elementary components.

In principle, an exact determination of the characteristics of passive components like transmission lines, resonators, and filters is possible by means of complete Fourier series expansions. For numerical evaluation, only a finite number of terms can be taken into account. Hence, this method is characterized by the fact that the exactly formulated problem is solved approximately.

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The authors are with the Department of Electrical Engineering, Fernuniversitaet, Hagen, Federal Republic of Germany.

A completely different way is taken by the grid-point method and the method of lines [1], where the approximately formulated problem is solved exactly.

The semi-analytical method of lines has been applied to various problems of physics [2]. An essential extension of this method is given in [3] for the one-dimensional and in [4] for the two-dimensional hybrid problem of planar waveguides. It has been shown that this class of waveguides can be solved accurately and in a simple manner.

In the limiting case of an infinite number of lines, exactly the same solution is obtained as in the limiting case of an infinite number of terms in the Fourier series expansions.

The relative convergence phenomenon, which is a consequence of the Fourier series truncations, does not occur with the method of lines. Optimum convergence is always assured, if the simple condition is satisfied that the strip-edges are located at definite positions with respect to the adjacent ψ^e- and ψ^h-lines [5]. It should be noted, however, that the convergence of the propagation constant, the characteristic impedance or the resonant frequency does not critically depend on the edge parameters, so that the problem of convergence on the whole is not critical.

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This is the main advantage of the method of lines for planar structures.

In order to satisfy correctly the edge condition for each edge of a given waveguide and to satisfy in addition the lateral boundary conditions, an appropriate number of lines has to be determined. It is obvious that this problem becomes more difficult with an increasing number of conductors. A further deficiency of the method is given by the fact that, in case of extreme differences in the widths of the conductors and the spacings between them, the total number of lines increases considerably.

The reason for these drawbacks lies in the inflexibility of the equidistant discretization.

In the present paper, it will be shown that the nonequidistant discretization, which has been applied successfully in the grid-point method, can also be introduced in the method of lines without changing its special transformation properties. An outline of the method will be given for the one-dimensional nonequidistant discretization. The extension of this method to two-dimensional problems does not cause any difficulties: the procedure is similar to that given in [4].

Numerical results are presented for two selected examples: the coplanar waveguide (one-dim. discretization) and the hair-pin resonator (two-dim. discretization). The convergence behavior is discussed and comparisons are made with the limiting case of equidistant discretization.

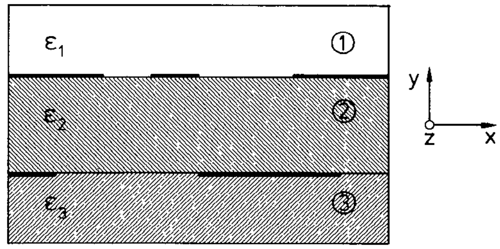

Fig. 1. Cross section of a planar microwave structure.

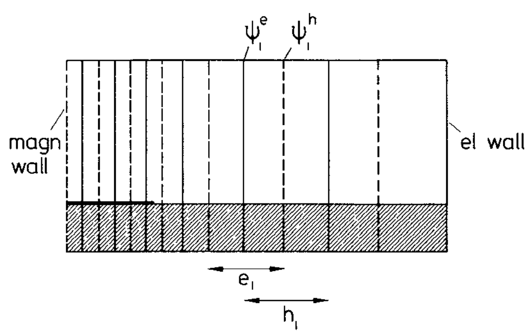

Fig. 2. Position of the discretization lines for the scalar potentials ψ^e and ψ^h; e_i and h_i designate interval sizes.

II. Formulation

The cross-section of the structure is subdivided into several partial areas, as indicated in Fig. 1. Within each area, constant permittivity is assumed. Conducting strips of vanishing thickness are located at the interfaces between the areas.

The electromagnetic field components $\vec{E}$ and $\vec{H}$ are derived from two independent vector potential functions, which in each case exhibit only one component in z-direction

$$\vec{E} = \nabla \times \nabla \times (\Psi^e \vec{e}_z)/j\omega\epsilon - \nabla \times (\Psi^h \vec{e}_z) \quad (1)$$

$$\vec{H} = \nabla \times (\Psi^e \vec{e}_z) + \nabla \times \nabla \times (\Psi^h \vec{e}_z)/j\omega\mu_0. \quad (2)$$

The harmonic time dependence $\exp(j\omega t)$ has been omitted for brevity.

For waveguides uniform in the direction of propagation (z-direction), the two scalar functions of the vector potentials can be expressed as

$$\Psi^{e,h} = \psi^{e,h}(x, y)\exp(-j\beta z) \quad (3)$$

where β is the propagation constant.

Substituting (3) in the corresponding Helmholtz equations for the scalar potential functions yields

$$\frac{\partial^2 \psi^{e,h}}{\partial y^2} + \frac{\partial^2 \psi^{e,h}}{\partial x^2} + (k^2 - \beta^2)\psi^{e,h} = 0 \quad (4)$$

with $k^2 = \omega^2 \mu_0 \epsilon_0 \epsilon$.

The potential functions are submitted to homogeneous Dirichlet or Neumann conditions on the shielding (and symmetry) walls. Continuity conditions have to be satisfied at the boundaries between the different areas.

Because of strip-conductor edges, the electromagnetic fields exhibit singularities. Hence, a discrete representation is chosen along the interfaces (x-direction), whereas in the vertical direction the fields are expressed analytically. This means that the potential functions ψ^e and ψ^h are considered on lines, as illustrated in Fig. 2.

The shifting of the two sets of lines with respect to each other is a necessary condition for the compatibility of the operators applied in the following. As a consequence of the shifting, both the lateral boundary conditions and the edge condition fit in harmoniously.

The sizes of the intervals intersected by the discretization lines for ψ_i^e and ψ_i^h are denoted by e_i $(i = 1, \cdots, N_e)$ and h_i $(i = 1, \cdots, N_h)$, respectively.

In order to obtain symmetric second-order operators, normalized potential functions are introduced next

$$\phi_i^e = \sqrt{e_i/h}\,\psi_i^e \quad (5a)$$

and

$$\phi_i^h = \sqrt{h_i/h}\,\psi_i^h \quad (5b)$$

where h represents the interval size of the limiting case of equidistant discretization.

In matrix notation, (5a) and (5b) lead to the following equations:

$$\vec{\psi}^e = [r_e]\vec{\phi}^e \quad (6a)$$

and

$$\vec{\psi}^h = [r_h]\vec{\phi}^h \quad (6b)$$

with

$$[r_e] = \mathrm{diag}\left(\sqrt{h/e_i}\right), \qquad [r_h] = \mathrm{diag}\left(\sqrt{h/h_i}\right). \quad (7)$$

It should be noted that the vectors and the matrices with the subscripts e and h are of order N_e and N_h, respectively. The finite-difference expression for the first derivative of ψ^e with respect to the x-direction is evaluated on the discretization line for ψ^h. Hence, on the line for ψ_i^h, marked in Fig. 2, the first derivative of ψ^e is approximated as follows:

$$\left.\frac{\partial \psi^e}{\partial x}\right|_i \simeq \frac{\psi_{i+1}^e - \psi_i^e}{h_i}. \tag{8}$$

After normalization, this becomes

$$\sqrt{h_i/h}\left(h\left.\frac{\partial \psi^e}{\partial x}\right|_i\right) \simeq \sqrt{h/h_i}\,(\psi_{i+1}^e - \psi_i^e) \tag{9}$$

or, in matrix notation

$$\begin{aligned}[r_h]^{-1}\left(h\,\overrightarrow{\frac{\partial \psi^e}{\partial x}}\right) &\longrightarrow [r_h][D]\vec{\psi}^e \\ &= [r_h][D][r_e]\vec{\phi}^e \\ &= \quad [D_x]\vec{\phi}^e. \end{aligned} \tag{10}$$

In the case of equidistant discretization, characterized by the relation $h_i = e_i = h$ for all i, the bidiagonal matrix $[D_x]$ is identical to the difference operator $[D]$, which is given in [4] for the various combinations of lateral boundary conditions. For the combination magnetic/electric wall of Fig. 2, one obtains the following square matrix:

$$[D] = \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & -1 \end{bmatrix}. \tag{11}$$

On account of the dual lateral boundary conditions and the shifting of lines, the finite-difference translation for the first derivative of ψ^h can be given immediately

$$[r_e]^{-1}\left(h\,\overrightarrow{\frac{\partial \psi^h}{\partial x}}\right) \longrightarrow -[r_e][D]^t[r_h]\vec{\phi}^h = -[D_x]^t\vec{\phi}^h. \tag{12}$$

Combining the first-order operators, one obtains for the second-order derivatives

$$[r_e]^{-1}\left(h^2\,\overrightarrow{\frac{\partial^2 \psi^e}{\partial x^2}}\right) = h^2\,\overrightarrow{\frac{\partial^2 \phi^e}{\partial x^2}} \longrightarrow -[D_x]^t[D_x]\vec{\phi}^e \tag{13a}$$

$$[r_h]^{-1}\left(h^2\,\overrightarrow{\frac{\partial^2 \psi^h}{\partial x^2}}\right) = h^2\,\overrightarrow{\frac{\partial^2 \phi^h}{\partial x^2}} \longrightarrow -[D_x][D_x]^t\vec{\phi}^h. \tag{13b}$$

The second-order operators

$$\begin{aligned}[D_{xx}^e] &= -[D_x]^t[D_x] \\ [D_{xx}^h] &= -[D_x][D_x]^t \end{aligned} \tag{14}$$

are real-symmetric tridiagonal matrices. Thus, they can be transformed by orthogonal transformation into the diagonal form of their real and distinct eigenvalues

$$[T_e]^t[D_{xx}^e][T_e] = [\lambda^e]$$

and

$$[T_h]^t[D_{xx}^h][T_h] = [\lambda^h] \tag{15}$$

where $[T_e]$ and $[T_h]$ are the matrices of the eigenvectors. It can be proved that the bidiagonal first-order operator $[D_x]$ is transferred to quasi-diagonal form by the following transformation [6]:

$$[T_h]^t[D_x][T_e] = [\delta]. \tag{16}$$

From (14) to (16), the following relations are derived:

$$[\lambda^e] = -[\delta]^t[\delta]$$

and

$$[\lambda^h] = -[\delta][\delta]^t. \tag{17}$$

In case of different lateral boundary conditions (magn. wall/el. wall, and vice versa) $[\delta]$ is a square diagonal matrix and (17) is reduced to

$$[\lambda^e] = [\lambda^h] = -[\delta]^2. \tag{18}$$

The eigenvalues and the matrices of the eigenvectors in (15) are determined by means of the 'Implicit QL-method' [7], an accurate and numerically stable method.

Only in the limiting case of equidistant discretization, these quantities are given in analytical form.

The partial differential equations (4) can now be transferred to the following systems of ordinary differential equations:

$$\frac{d^2\vec{V}^{e,h}}{dy^2} + \left([\lambda^{e,h}]/h^2 + (k^2 - \beta^2)\right)\vec{V}^{e,h} = 0 \tag{19}$$

with $\vec{V}^{e,h} = [T_{e,h}]^t\vec{\phi}^{e,h}$. The solutions V_j^e and V_j^h, respectively, of these one-dimensional Helmholtz equations correspond to the simple transmission line equations.

The boundary conditions at the top and the bottom shielding, as well as the matching of the fields at the interfaces, can be carried out using only diagonal matrices. An inhomogeneous matrix equation is obtained:

$$[\tilde{Z}]\begin{bmatrix}\vec{\tilde{J}}_z \\ \vec{\tilde{J}}_x\end{bmatrix} = \begin{bmatrix}\vec{\tilde{E}}_z \\ \vec{\tilde{E}}_x\end{bmatrix} \tag{20}$$

where $(\vec{\tilde{J}}_z, \vec{\tilde{J}}_x)$ represents the transformed current distribution and $(\vec{\tilde{E}}_z, \vec{\tilde{E}}_x)$ the transformed tangential electric field at the interfaces.

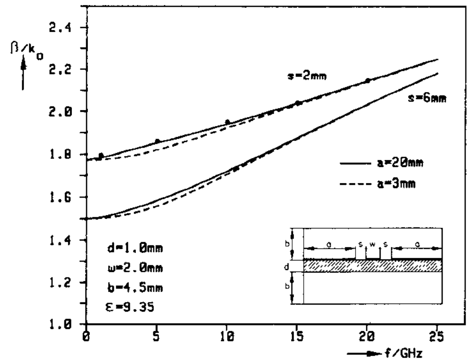

Fig. 3. Normalized phase constant β/k_0 versus frequency for the fundamental mode of the coplanar waveguide; ●... [8].

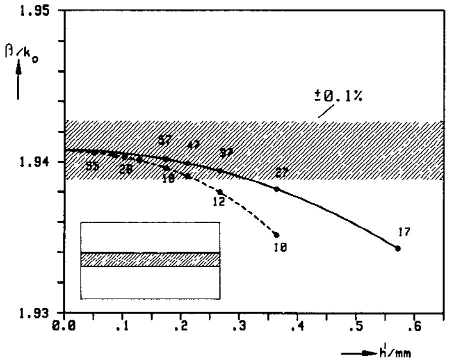

Fig. 4. Convergence behavior of the propagation constant as a function of the interval size h' for the center conductor; structural data as in Fig. 3, $f = 10$ GHz, $a = 7$ mm, $s = 2$ mm.

From (20), a reduced matrix equation in the original domain is derived:

$$[Z(\beta)]\begin{bmatrix}\vec{J}_z\\ \vec{J}_x\end{bmatrix}_{\text{strip}} = \begin{bmatrix}\vec{E}_z\\ \vec{E}_x\end{bmatrix}_{\text{strip}} = 0. \tag{21}$$

The propagation constants are found from the corresponding determinantal equation. By the Gaussian algorithm, one obtains the current distributions, which represent the current densities per interval. A simple multiplication yields the current density per unit length

$$J'_{zi} = e_i J_{zi}$$
$$J'_{xi} = h_i J_{xi}. \tag{22}$$

III. Results and Discussion

The method presented has been applied to the coplanar waveguide. In Fig. 3, the dispersion characteristics are given for different slot-widths and distances of the lateral shielding. As can be seen, the propagation constant is mainly determined by the slot-width. However, in the lower frequency range and for decreasing slot-width, the influence of the lateral shielding cannot be neglected.

The convergence behavior of the propagation constant as a function of the smallest interval size is illustrated in Fig. 4. Equidistant and nonequidistant discretization have the same curve of convergence (drawn curve), as long as

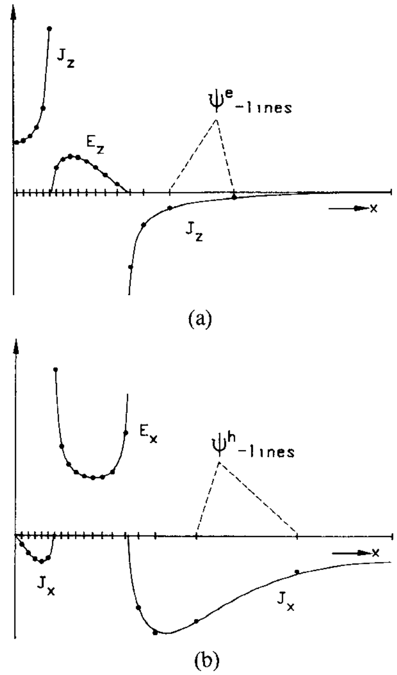

Fig. 5. Distribution of the electric field in the slot and the surface current on the strips at $f = 10$ GHz for the coplanar waveguide. The values from the nonequidistant discretization are given by dots ($N_e = 18$), those from the equidistant discretization are located on the drawn curves ($N_e = 57$); $J_z/J_x \simeq 14$, $E_x/E_z \simeq 14$.

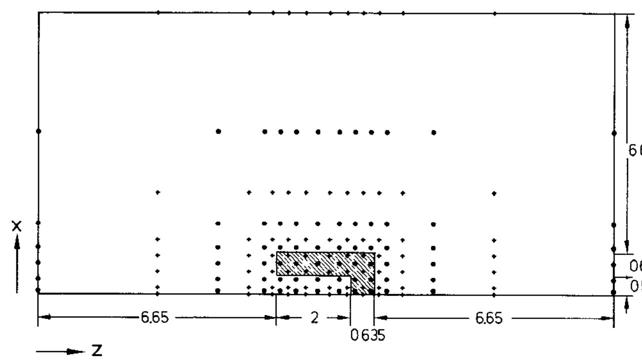

Fig. 6. Discretization lines for the hair-pin resonator (top view); + for ψ^e-lines, ● for ψ^h-lines; $h' \simeq 0.42$ mm (cf. Fig. 8).

the discretization between the symmetry wall and the outer slot-edge is equidistant. The discretization of the remaining distance 'a' has only little effect. However, if in the above-named region of high field concentration a nonequidistant discretization is chosen, the curve of convergence changes; for the dashed curve, the interval size at the outer edge is twice that near the symmetry wall. At the marked points, the total number of ψ^e-lines, needed for half the waveguide, is indicated. The numbers at the drawn curve refer to the equidistant case.

Finally, the distribution of the electric field in the slot and of the surface current on the strips is depicted in Fig. 5. The discretization corresponds to that of the dashed curve in Fig. 4. Near the strip-edges the fields vary rapidly, so that a fine discretization is chosen there. Exterior to these regions of high energy concentration, the functions are smooth. Hence, a coarse discretization is adequate. The

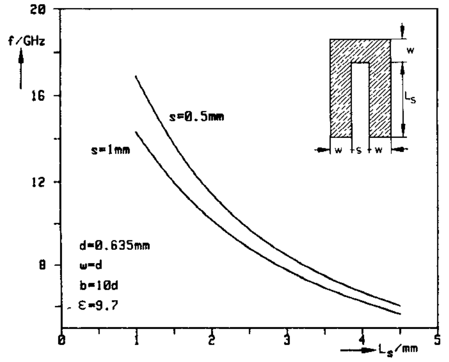

Fig. 7. Resonant frequency of a hair-pin resonator versus the stub length L_s for different spacings s.

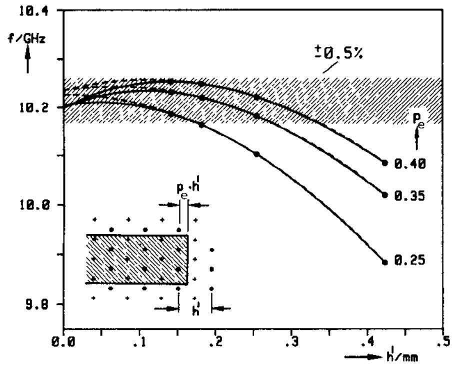

Fig. 8. Convergence behavior of the resonant frequency as a function of the interval size h' at the open end of the resonator; --- parabolic interpolation, ——— cubic interpolation; structural data as in Fig. 7, $L_s = 2$ mm, $s = 1$ mm.

tick-marks on the x-axis indicate the positions of the ψ^e-lines (Fig. 5(a)) and of the ψ^h-lines (Fig. 5(b)), respectively, which are shifted to each other. The corresponding values for the field components (dots) deviate only slightly from the more accurate values of the drawn curves, which are calculated by the equidistant discretization.

The advantages of nonequidistant discretization become particularly evident for two-dimensional problems. In Fig. 6, the pattern of lines for a hair-pin resonator is shown. As the results depend mainly on the accurate incorporation of the edges, fine and equidistant discretization is chosen in the vicinity of the contour. Exterior to the conductor, the fields are evanescent and smooth, so that a coarse discretization is advantageous. Inside the bordered region, 55 ψ^e-lines are located. The equidistant discretization, $h = h'$, would need 629 ψ^e-lines without giving more accurate results.

The computation time depends on the total number of lines, in particular on the number of lines that pass through the conducting structure and that determines the order of the reduced matrix (cf. (21)). By nonequidistant discretization, the computational effort can be reduced by a factor $4 \cdots 10$, compared with the effort in case of equidistant discretization according to [4].

For different spacings 's' the resonant frequency as a function of the stub length is shown in Fig. 7.

Fig. 8 gives the convergence behavior of the resonant frequency. As illustrated in the detail, the parameter 'p_e' determines the spacing between the ψ^h-lines and the conductor at the open end of the resonator. The (marked) results that correspond to $h' \simeq 0.42$ mm are obtained with the pattern of lines from Fig. 6. For h' less than 0.17, all results are within the margins of error of 0.5 percent. It has to be emphasized that an equidistant discretization would, in this range of interval sizes, overstress even large computers.

The two sets of curves represent parabolic (dashed) and cubic (drawn) curves of interpolation, respectively. In case of parabolic interpolation, for each 'p_e' only three values of the interval size h' are taken into account. For verification, a fourth value is calculated with finer discretization. As can be recognized, these points are located fairly well on the corresponding curves of interpolation. The extrapolated results of the dashed curves are in good agreement.

In case of cubic interpolation, all marked points of each 'p_e'-value are included. The extrapolated results, which are also in good agreement, differ less than 0.5 percent from those of the parabolic curves.

IV. Hints for Computation

As has been mentioned above, the discretization should follow the change of the field components. In the vicinity of conductor-edges, equidistant and fine discretization is adequate. Exterior to these regions, the interval sizes may increase. In order to obtain a uniform error distribution for the two scalar potentials, successive interval sizes should not differ too much.

This is easily achieved, if at first the sub-intervals between the ψ^e-lines and the ψ^h-lines are determined, e.g., as a geometrical series where the quotient of successive sub-intervals is a constant 'q', and after that the ψ^e-lines and the ψ^h-lines are assigned alternately. Each interval size, e_i or h_i, is composed of two sub-intervals.

The advantages of the geometrical series are easily shown: if the conductor-width of the coplanar waveguide (detail of Fig. 3) is designated by 'a' and the size of the sub-interval at the strip-edge by 'h_e', the following relation holds:

$$a = h_e \frac{q^M - 1}{q - 1}$$

where 'M' is the sum of the ψ^e- and the ψ^h-lines between the edge and the lateral shielding.

The quotient 'q' should not exceed the range $1 < q < 1.5$.

V. Conclusions

The method of lines as presented in this paper is highly adapted to planar waveguide problems. As the accuracy of the solutions mainly depends on the incorporation of the fields near the strip-conductor edges, the discretization should be fine in these regions. Exterior to the contours the fields are smooth, so that a coarse discretization is adequate. A development of the method to the inhomogeneous (source-type) waveguide problem, as presented in [9] for equidistant discretization, is possible. The difficulty of positioning the sources in a sufficiently large distance from discontinuities is reduced considerably.

In principle, the method presented includes the possibility for calculating planar stripline structures, where the permittivity of the substrate is given by $\epsilon = \epsilon(x)$. In that case, the partial differential equations for the scalar potentials are of the Sturm–Liouville type [6].

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Heinrich Diestel was born in Haselünne, Germany, on April 16, 1952. He received the Dipl.-Ing. degree from the Technical University in Hannover, Germany, in 1978 and the Dr.-Ing. degree from the Fernuniversitaet in Hagen, Germany, in 1984.

Since 1979, his research activities have been in the area of planar waveguides for integrated optics and planar microwave structures.

Stephan B. Worm was born in Bladel, the Netherlands, in 1951. He received the M.Sc. degree from the Eindhoven University of Technology, Eindhoven, the Netherlands, in 1978, and the Ph.D. degree from the Fernuniversitaet in Hagen, Germany, in 1983.

From 1978 to 1983, he was employed at the Fernuniversitaet, Hagen, where he was engaged in theoretical investigations of the properties of planar microwave structures. In 1983, he joined Philips, Elcoma Division, where he is engaged in the development of microwave tubes.

Short Papers

High-Order Mode Cutoff In Rectangular Striplines

CLAUDE M. WEIL, MEMBER, IEEE, AND LUCIAN GRUNER, MEMBER, IEEE

Abstract —**The higher order mode cutoff characteristics of rectangular stripline structures, with thin center conductors, are discussed. Data are given, using an alternative method of presentation, on the normalized cutoff of the first eleven higher order modes. Discussions are included on the physical reasons why cutoff is altered for some modes, relative to that in rectangular waveguides, but not for others.**

I. Introduction

Large-scale rectangular strip-transmission lines containing a propagating transverse electromagnetic (TEM) field are now widely used for such purposes as electromagnetic susceptibility and emissions testing, calibration of field probes and survey meters, and studies on the biological effects of radiofrequency (RF) radiation exposure. These structures are characterized by an air dielectric and a thin center conductor (septum) surrounded by a rectangularly shaped shield. This provides for an optimally sized test space within the line in which equipment, field probes, or experimental animals, etc., are exposed to a well-defined and reasonably uniform field. Crawford [1] has discussed the properties of such lines and has described a family of TEM "cells" constructed at the National Bureau of Standards. These devices are commercially available and have been termed "Crawford Cells" or "TEM Transmission Cells" by their manufacturers.

The usable frequency range of these devices is of obvious importance to those involved in their use. Whereas it had been thought that these structures could not be used above the cutoff frequency where the first higher order mode is predicted to occur [2], it has recently been shown by Hill [3] that such is not necessarily the case. In his important study, Hill has shown that significant perturbation of the internal fields within the structure exists primarily at certain discrete frequencies where resonances of the higher order mode fields occur. Such resonances will occur when the equivalent electrical length of the strip-transmission line

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C. M. Weil is with the Boeing Military Airplane Company, Mail Stop 40-35, P.O. Box 3707, Seattle, WA 98124.

L. Gruner is with the Department of Electrical Engineering, Monash University, Clayton, Victoria, Australia 3168.